International Design Study for the Neutrino Factory
10th Plenary Meeting

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New developments in leptonic flavour studies

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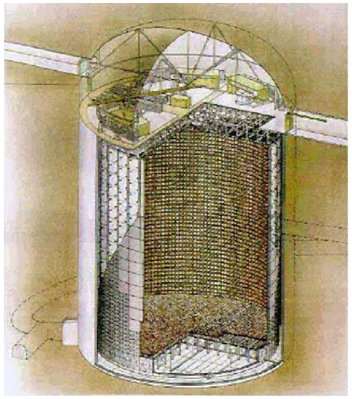
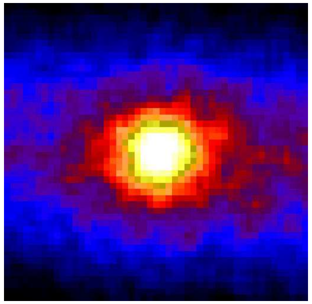
Neutrino oscillations: experimental milestones



- atmospheric neutrinos
  - ν_μ / $\overline{\nu}_\mu$ disappear – Super-Kamiokande (1998)
- accelerator neutrinos
  - ν_μ disappear – K2K (2002), MINOS (2006)
  - ν_μ converted to ν_τ – OPERA (2010 & 2012)
  - ν_μ converted to ν_e – T2K, MINOS (2011)



- solar neutrinos
  - ν_e disappear – Chlorine (1998), Gallium (1999 - 2009), Super-Kamiokande (2002), Borexino (2008)
  - ν_e converted to $(\nu_\mu + \nu_\tau)$ – SNO (2002)
- reactor neutrinos
  - $\overline{\nu}_e$ disappear – Double Chooz (2011), Daya Bay, RENO (2012)
  - $\overline{\nu}_e$ disappear – KamLAND (2002)

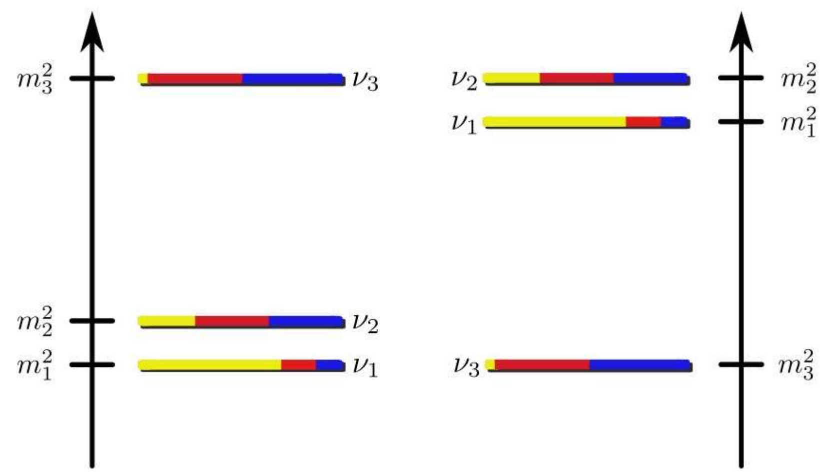
Three neutrino flavour mixing

for steriles $\rightarrow$ Merle & Bayes

(in diagonal charged lepton basis)

flavour PMNS mixing mass

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3}\\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$



atmospheric reactor + Dirac solar Majorana

$$U_{\text{PMNS}} = \begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & 0 & s_{13}e^{-i\delta}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta} & 0 & c_{13}\end{pmatrix}\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & e^{\frac{\alpha_2}{2}} & 0\\ 0 & 0 & e^{\frac{\alpha_3}{2}}\end{pmatrix}$$

$\theta_{23} \approx 45^\circ$ $\theta_{13} \approx 9^\circ$ $\theta_{23} \approx 34^\circ$

Fermion mixings

- mismatch of flavour (weak) and mass eigenstates

$$\Psi_{\text{flavour}} = V^\dagger \, \Psi_{\text{mass}}$$

- quark sector: V_L^u and V_L^d

$$U_{\text{CKM}} = V_L^u \, V_L^{d\dagger} \approx \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \qquad \lambda \sim 0.22$$

- lepton sector: V_L^e and V_L^ν

$$U_{\text{PMNS}} = V_L^e \, V_L^{\nu\dagger} \approx \begin{pmatrix} 0.8 & 0.5 & 0.2 \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix}$$

Gonzalez-Garcia et al. (2012)

mixing $\Longleftrightarrow$ each family knows of the existence of the others!

Non-Abelian family symmetries

Dynamical origin of Yukawa couplings

- models of flavour are typically formulated at high energies
- separate family symmetry from EW symmetry breaking

M_{Pl}

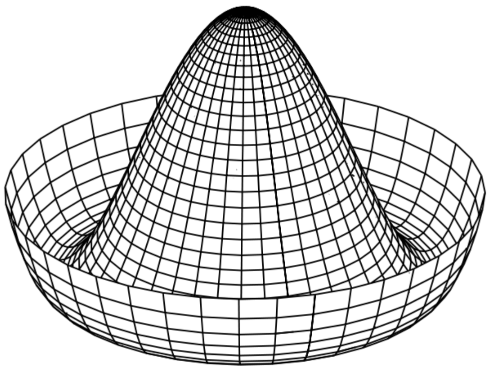
flavon VEV $\langle \phi \rangle$ breaks family symmetry

M_{GUT} $\quad \rightarrow$ Yukawas Y_{ij}

M_{seesaw}

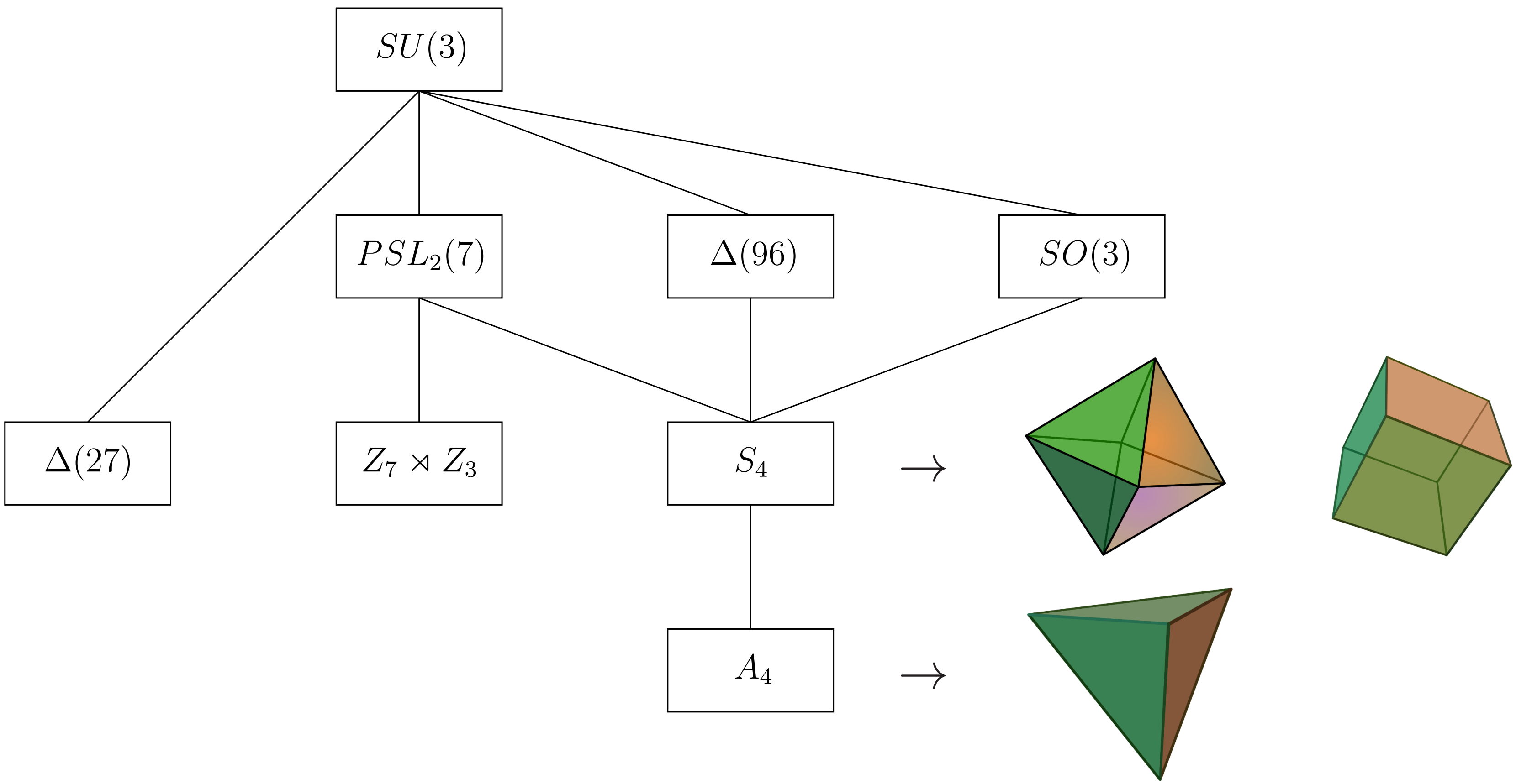
Higgs VEV $\langle H \rangle$ breaks electroweak symmetry

M_{EW} $\quad \rightarrow$ fermion masses $M_{ij} = Y_{ij} \langle H \rangle$



Candidate symmetries

- unify three families in multiplets of family symmetry
- underlying group should have three-dimensional representations



Tri-bimaximal lepton mixing vs. global neutrino fits

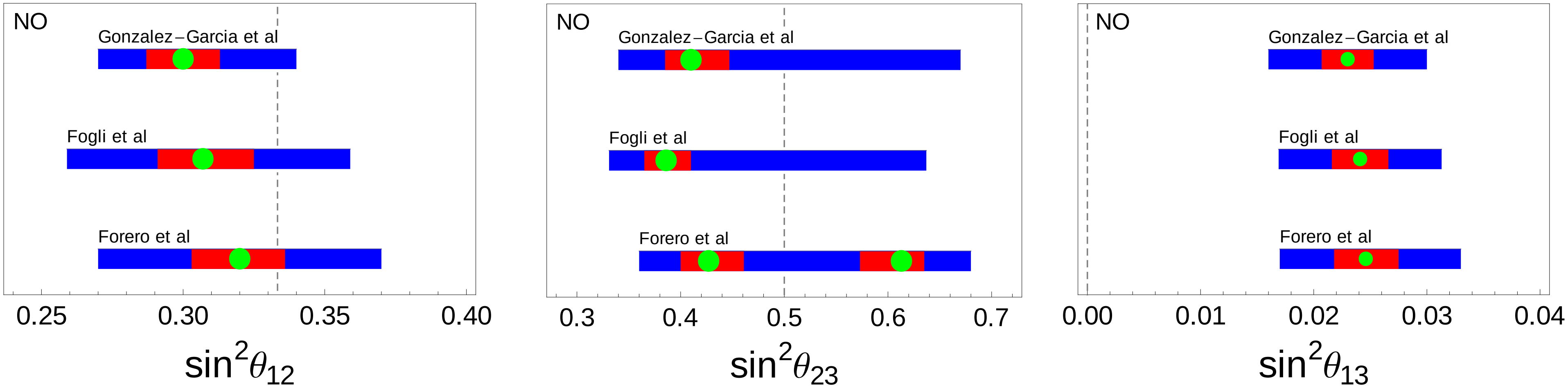
- A_4 and S_4 most popular candidates until recently
- natural symmetries for tri-bimaximal (TB) mixing

$$U_{\rm PMNS} \approx U_{\rm TB} \equiv \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$


Harrison


Perkins


Scott



$_{\text{(exact)}}$ TB mixing is dead!

T2K [arXiv:1106.2822]

- $\theta_{13} \neq 0$ disfavoured at $\sim 2.5\sigma$
- $5° \lesssim \theta_{13} \lesssim 18°$ at 90% C.L.

Double Chooz [arXiv:1207.6632]

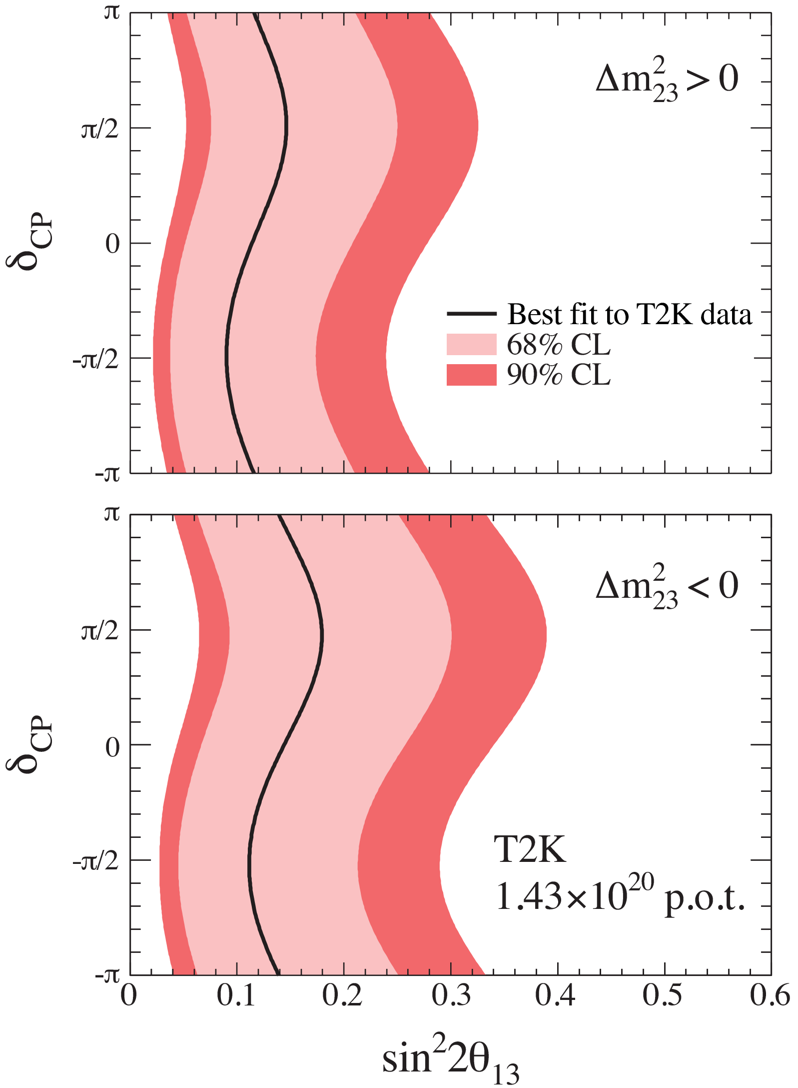
- $\theta_{13} \neq 0$ disfavoured at $\sim 2.9\sigma$
- $6° \lesssim \theta_{13} \lesssim 12°$ at 90% C.L.

RENO [arXiv:1204.0626]

- $\theta_{13} \neq 0$ disfavoured at $\sim 4.9\sigma$
- $8.0° \lesssim \theta_{13} \lesssim 11.4°$ at 90% C.L.

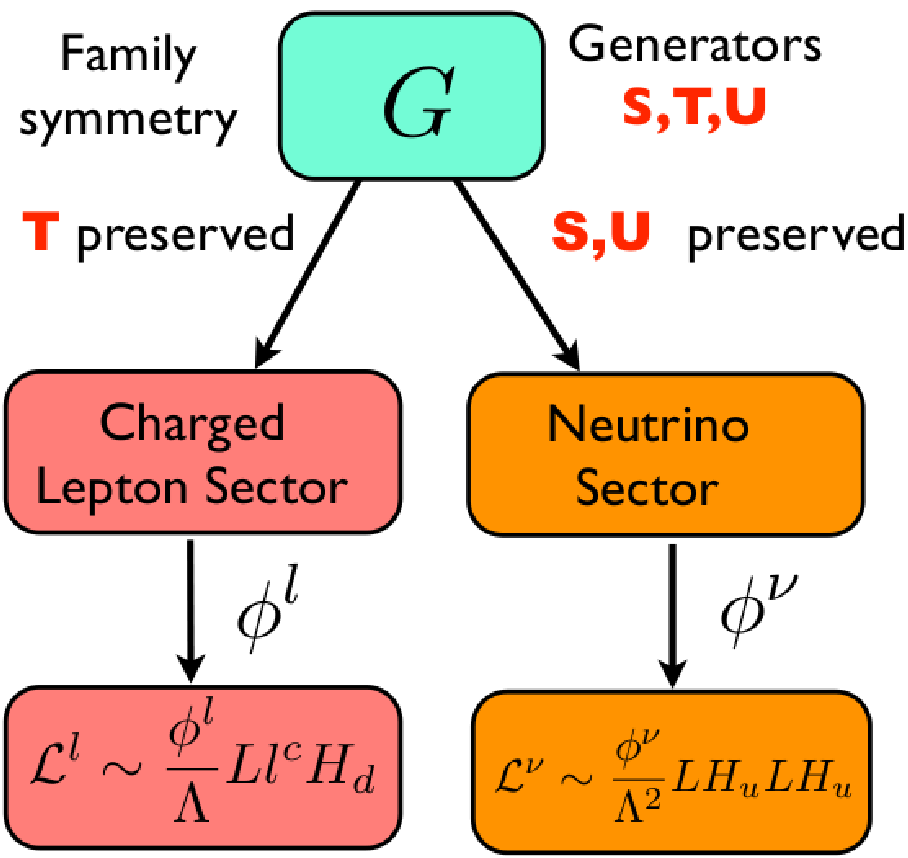
Daya Bay [arXiv:1210.6327]

- $\theta_{13} \neq 0$ disfavoured at $\sim 7.7\sigma$
- $7.7° \lesssim \theta_{13} \lesssim 9.6°$ at 90% C.L.

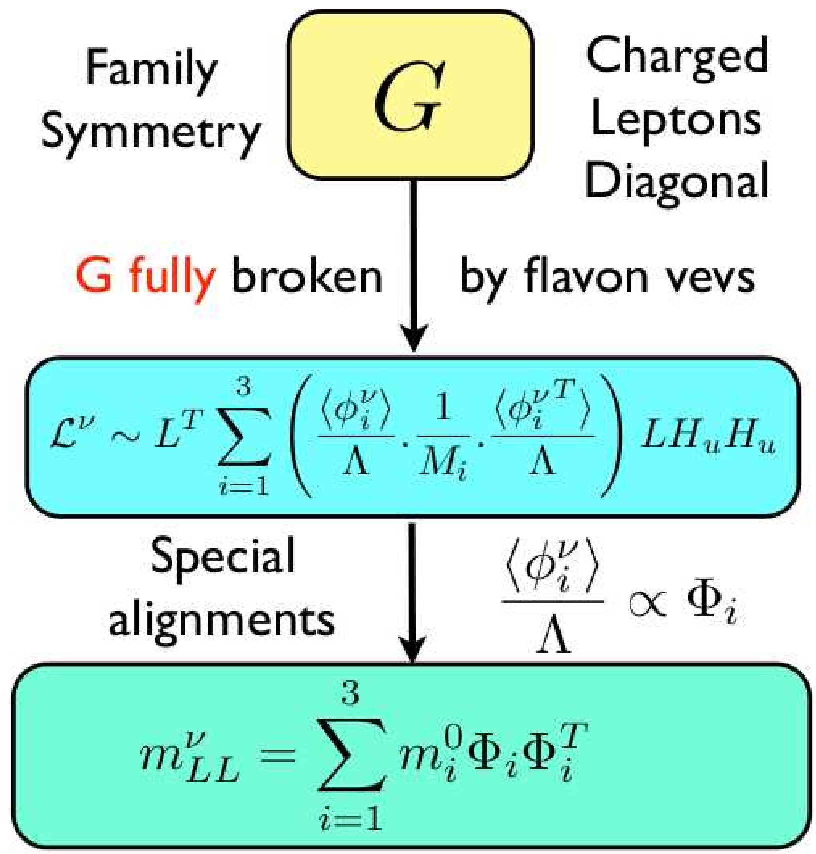


The (simple) good old days are gone!

Two model building approaches



(direct models)



(indirect models)

The future of family symmetries

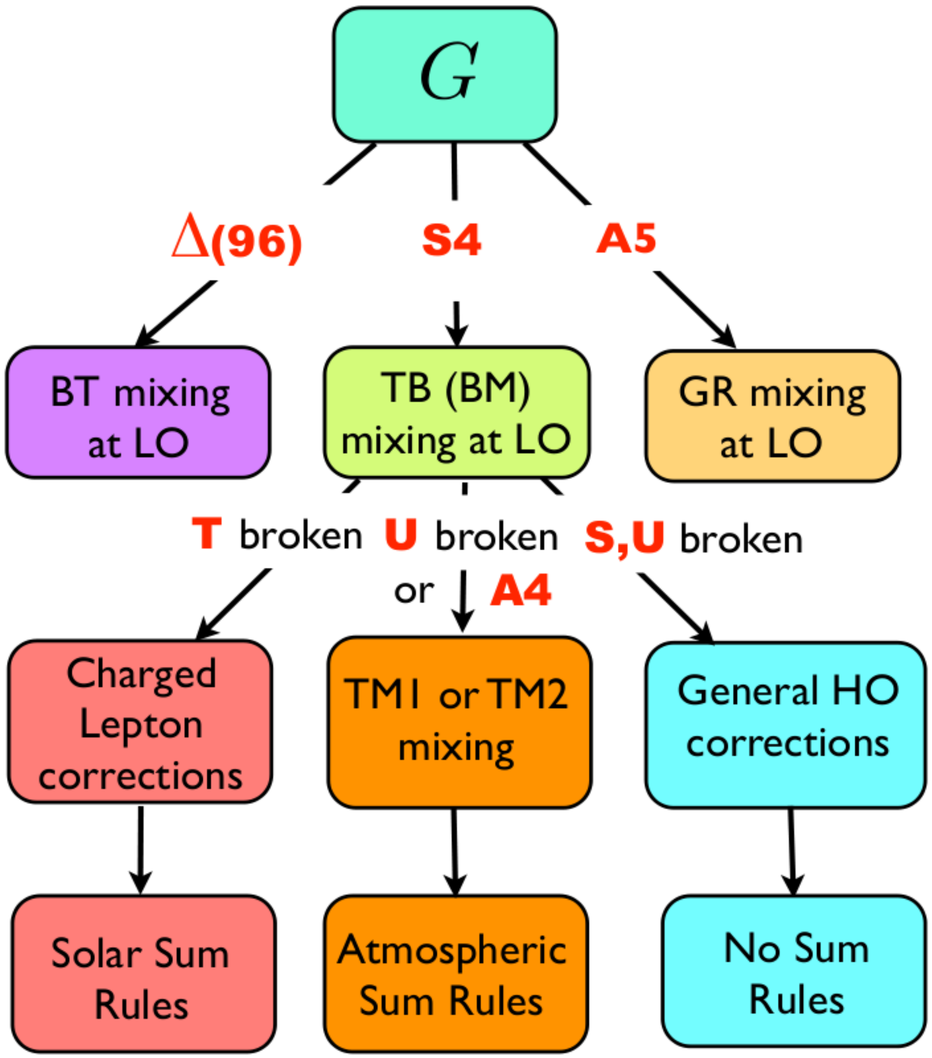
King, Luhn (arXiv:1301.1340)

PMNS mixing follows from

↙ ↘

symmetry	flavon VEVs
direct approach	indirect approach
TB plus corrections	TB plus corrections
non-standard family symmetries	non-standard flavon VEV configurations

Direct models after Daya Bay and RENO



mixing patterns:

	θ_{13}	θ_{23}	θ_{12}
TB	$0°$	$45°$	$35.3°$
BM	$0°$	$45°$	$45°$
GR	$0°$	$45°$	$31.7°$
BT	$12.2°$	$36.2°$	$36.2°$
TM	$\neq 0°$	$\neq 45°$	$35.3°$

TB = tri-bimaximal
BM = bimaximal
GR = golden ratio
BT = bi-trimaximal
TM = trimaximal

TB plus corrections (in S_4)

Breaking the T symmetry – I

- charged lepton mass matrix only approximately diagonal
- $U_{\rm PMNS} = V_{\ell_L} V_{\nu_L}^\dagger$
- $V_{\ell_L}^\dagger = \underbrace{U_{23}^{\ell_L} U_{13}^{\ell_L} U_{12}^{\ell_L}}_{\text{small mixing}}$ from M_ℓ and $V_{\nu_L}^\dagger = \underbrace{U_{23}^{\nu_L} U_{13}^{\nu_L} U_{12}^{\nu_L}}_{U_{TB}}$ from M_ν

$$U_{\rm PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & \hat{s}_{23} \\ 0 & -\hat{s}_{23}^* & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & \hat{s}_{13} \\ 0 & 1 & 0 \\ -\hat{s}_{13}^* & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & \hat{s}_{12} & 0 \\ -\hat{s}_{12}^* & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$s_{12}\, e^{i\delta_{12}} \approx \frac{1}{\sqrt{3}} \left(e^{i\delta_{12}^\nu} - \theta_{12}^\ell\, e^{i\delta_{12}^\ell} + \theta_{13}^\ell\, e^{i(\delta_{13}^\ell - \delta_{23}^\nu)} \right)$$

$$s_{23}\, e^{i\delta_{23}} \approx \frac{1}{\sqrt{2}} \left(e^{i\delta_{23}^\nu} - \theta_{23}^\ell\, e^{i\delta_{23}^\ell} \right)$$

$$s_{13}\, e^{i\delta_{13}} \approx \frac{1}{\sqrt{2}} \left(-\theta_{12}^\ell\, e^{i(\delta_{12}^\ell + \delta_{23}^\nu)} - \theta_{13}^\ell\, e^{i\delta_{13}^\ell} \right)$$

$$c_{ij} = \cos\theta_{ij}$$

$$\hat{s}_{ij} = \sin\theta_{ij}\, e^{-i\delta_{ij}}$$

Breaking the T symmetry – II

- if dominant contribution from θ^ℓ_{12}

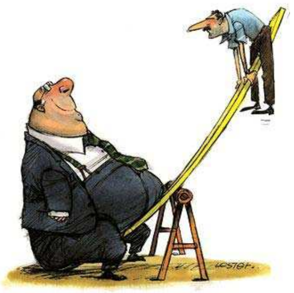
$$\begin{aligned}\theta_{13} &\approx \tfrac{1}{\sqrt{2}}\,\theta^\ell_{12}\\ \theta_{23} &\approx 45^\circ\\ \theta_{12} &\approx 35.3^\circ + \theta_{13}\cos\delta \qquad \text{(solar sum rule)}\end{aligned}$$

- dial one angle: $\theta^\ell_{12} \;\rightarrow\; \theta_{13}$
- still two predictions (relations for physical parameters)

- numerically intriguing: $\theta^\ell_{12} \approx \theta_C$
- simple GUT scenarios (Georgi-Jarlskog) predict $\theta^\ell_{12} \approx 4^\circ$
- sizable θ^ℓ_{12} might lead to a disfavoured solar angle θ_{12}
- special phase δ necessary (potentially predictable with CP symmetry)

Breaking the U symmetry

- diagonal charged leptons
- neutrino sector: tri-bimaximal at leading order



$$W_\nu \;\sim\; L N^c H_u + (\phi_{\mathbf{3'}} + \phi_{\mathbf{2}} + \phi_{\mathbf{1}}) N^c N^c + \frac{1}{M} \widetilde{\phi}_{\mathbf{1'}} \phi_{\mathbf{2}} N^c N^c$$

- flavon VEVs with alignments $\langle \phi_{\mathbf{3'}} \rangle = \varphi_{\mathbf{3'}} \,(1,1,1)^T \qquad \langle \phi_{\mathbf{2}} \rangle = \varphi_{\mathbf{2}} \,(1,1)^T$

$$U_{\text{PMNS}} \;\approx\; \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{6}}\alpha^* \\ \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{2}}\alpha & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}\alpha^* \\ \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{2}}\alpha & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}\alpha^* \end{pmatrix}$$

$\theta_{13} \approx 9^\circ$
requires
$|\alpha| \sim 0.2$

- S symmetry still preserved $\;\rightarrow\;$ trimaximal mixing (TM2)

$$\theta_{12} \;\approx\; 35.3^\circ$$
$$\theta_{23} \;\approx\; 45^\circ - \frac{1}{\sqrt{2}}\,\theta_{13} \cos\delta$$

(atmospheric sum rule)

generalisation & testability $\;\rightarrow\;$ Ballet

Non-standard family symmetries

Bi-trimaximal mixing from $\Delta(96)$

charged leptons $T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{4\pi i}{3}} & 0 \\ 0 & 0 & e^{\frac{2\pi i}{3}} \end{pmatrix}$

neutrinos $S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$ $\quad U = -\frac{1}{3}\begin{pmatrix} 1 & 1+\sqrt{3} & 1-\sqrt{3} \\ 1+\sqrt{3} & 1-\sqrt{3} & 1 \\ 1-\sqrt{3} & 1 & 1+\sqrt{3} \end{pmatrix}$

$\rightarrow$ generators T and S are common with S_4

$\rightarrow$ all mixing angles determined

$$U_{\mathrm{PMNS}} = U_{\mathrm{BT}} = \begin{pmatrix} -a_+ & \frac{1}{\sqrt{3}} & a_- \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ a_- & \frac{1}{\sqrt{3}} & -a_+ \end{pmatrix} \qquad a_\pm = \frac{1}{2} \pm \frac{1}{2\sqrt{3}}$$

$$\theta_{13} = 12.2^\circ \qquad \theta_{23} = 36.2^\circ \qquad \theta_{12} = 36.2^\circ$$

$\Longrightarrow$ corrections still essential in this case

Model predictions for CP violation

Combining family and CP symmetry

- theory symmetric under $\mathcal{G}$ transformation

$$\psi \xrightarrow{\mathcal{G}} \rho(g)\psi$$

- generalised CP transformation

$$\psi \xrightarrow{\text{CP}} X\psi^*$$

- sequence of transformations $\text{CP} \circ \mathcal{G} \circ (\text{CP})^{-1}$ is again $\mathcal{G}$ transformation

$$\psi \xrightarrow{\text{CP}} X\psi^* \xrightarrow{\mathcal{G}} X\rho^*(g)\psi^* \xrightarrow{(\text{CP})^{-1}} X\rho^*(g)X^{-1}\psi$$

- find all X which satisfy consistency condition $\boxed{X\rho^*(g)X^{-1} = \rho(g')}$
- these matrices form a representation of the automorphism group of $\mathcal{G}$
- $\text{Aut}(\mathcal{G})$ tabulated for finite groups $\mathcal{G}$
- $\mathcal{G} = S_4$: $X = \rho(h)$ with $h \in S_4$

Holthausen, Lindner, Schmidt (arXiv:1211.6953)
Feruglio, Hagedorn,Ziegler (arXiv:1211.5560)

S_4 model with broken U symmetry

- CP symmetry $\rightarrow$ all coupling constants real

$$W_\nu \;=\; y\,LN^cH_u + (y_{\mathbf{3'}}\,\phi_{\mathbf{3'}} + y_{\mathbf{2}}\,\phi_{\mathbf{2}} + y_{\mathbf{1}}\,\phi_{\mathbf{1}})N^cN^c + \frac{y_{\mathbf{1'}}}{M}\widetilde{\phi}_{\mathbf{1'}}\phi_{\mathbf{2}}N^cN^c$$

- spontaneous CP violation
- correlations between flavon VEVs from flavon potential

$$\varphi_{\mathbf{2}} = A\,\varphi_{\mathbf{1}} \qquad \varphi^2_{\mathbf{3'}} = B\,\varphi^2_{\mathbf{1}} \qquad \widetilde{\varphi}^2_{\mathbf{1'}} = C\,M^2$$

- phases of $\varphi_{\mathbf{1}}$, $\varphi_{\mathbf{2}}$, $\varphi_{\mathbf{3'}}$ identical up to π or $\pm\pi/2$

FOUR CASES

	θ_{13}	θ_{23}	θ_{12}	δ	(α_1, α_2)
(i)	free	$45^\circ \mp \frac{1}{\sqrt{2}}\theta_{13}$	35.3°	0 or π	0 or π
(ii)	free	45°	35.3°	$\pm\frac{\pi}{2}$	0 or π
(iii)	unphysical: two degenerate neutrino masses				
(iv)	unphysical: $\theta_{13} = 35.3^\circ$				

Conclusion

- non-Abelian symmetries can unify chiral families
- simplest mixing patterns with $\theta_{13} = 0$ ruled out
- new model building strategies
  - TB plus corrections $\rightarrow$ solar & atmospheric sum rules
  - new family symmetries, e.g. $\Delta(96)$
  - discussion of CP

accurate measurement of all three mixing angles
and the Dirac CP phase crucial to test these ideas

Thank you

Details of the direct S_4 model

matter	L	τ^c	μ^c	e^c	N^c	H_u	H_d
S_4	$\mathbf{3}$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$
Z_3^ν	1	2	2	2	2	0	0
Z_3^ℓ	0	2	1	0	0	0	0

King, Luhn (2011)

$$\langle \varphi_\ell \rangle = \begin{pmatrix} 0 \\ v_\ell \\ 0 \end{pmatrix} \qquad \langle \eta_\mu \rangle = \begin{pmatrix} 0 \\ w_\mu \end{pmatrix} \qquad \langle \eta_e \rangle = \begin{pmatrix} w_e \\ 0 \end{pmatrix}$$

flavons	φ_ℓ	η_μ	η_e	$\phi_{\mathbf{3}'}$	$\phi_{\mathbf{2}}$	$\phi_{\mathbf{1}}$	$\widetilde{\phi}_{\mathbf{1}'}$
S_4	$\mathbf{3}'$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{3}'$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}'$
Z_3^ν	0	0	0	2	2	2	0
Z_3^ℓ	1	1	2	0	0	0	0

$$\langle \phi_{\mathbf{3}'} \rangle = v_{\mathbf{3}'} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \langle \phi_{\mathbf{2}} \rangle = v_{\mathbf{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \langle \phi_{\mathbf{1}} \rangle = v_{\mathbf{1}} \qquad \langle \widetilde{\phi}_{\mathbf{1}'} \rangle = \widetilde{v}_{\mathbf{1}'}$$

Charged lepton sector

$$\boxed{W_\ell \;\sim\; \left[\tfrac{1}{M}(L\varphi_\ell)_{1'}\,\tau^c + \tfrac{1}{M^2}(L\varphi_\ell)_2\,\eta_\mu\,\mu^c + \tfrac{1}{M^2}(L\varphi_\ell)_2\,\eta_e\,e^c\right] H_d}$$

- Z_3^ℓ controls pairing of flavons with right-handed charged fermions
- different S_4 contractions of $(L\varphi_\ell)$ pick out different L_i components

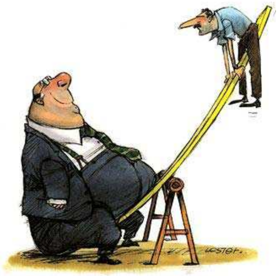
$$(L\varphi_\ell)_{1'} \;=\; L_1\varphi_{\ell 1} + L_2\varphi_{\ell 3} + L_3\varphi_{\ell 2} \quad\to\quad L_3$$

$$(L\varphi_\ell)_2 \;=\; \begin{pmatrix} L_1\varphi_{\ell 3} + L_2\varphi_{\ell 2} + L_3\varphi_{\ell 1} \\ L_1\varphi_{\ell 2} + L_2\varphi_{\ell 1} + L_3\varphi_{\ell 3} \end{pmatrix} \quad\to\quad \begin{pmatrix} L_2 \\ L_1 \end{pmatrix}$$

- mass matrix diagonal by construction
- m_τ heavier than m_μ and m_e
- hierarchy between m_μ and m_e due to hierarchy of VEVs w_μ and w_e
- just a toy model of charged lepton sector (with GUTs off-diagonals)

Neutrino sector

- discrete family symmetry with 24 elements
- irreducible S_4 representations: $\mathbf{1} \quad \mathbf{1}' \quad \mathbf{2} \quad \mathbf{3} \quad \mathbf{3}'$
- diagonal charged leptons (enforced by T symmetry)
- in neutrino sector $\quad (L \sim N^c \sim \mathbf{3} \qquad H_u \sim \mathbf{1})$



$$\boxed{W_\nu \;\sim\; LN^cH_u + (\phi_{\mathbf{3}'} + \phi_{\mathbf{2}} + \phi_{\mathbf{1}})N^cN^c + \frac{1}{M}\widetilde{\phi}_{\mathbf{1}'}\phi_{\mathbf{2}}N^cN^c}$$

S_4 irrep	S	U	VEV alignment
$\mathbf{3}'$	$\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$	$\langle \phi_{\mathbf{3}'} \rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$
$\mathbf{2}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\langle \phi_{\mathbf{2}} \rangle \propto \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
$\mathbf{1}$	1	1	$\langle \phi_{\mathbf{1}} \rangle \propto 1$
$\mathbf{1}'$	1	-1	$\langle \widetilde{\phi}_{\mathbf{1}'} \rangle \propto 1$

$\longrightarrow$ TB Klein symmetry broken in U (at higher order)

Resulting neutrino mixing

$$U_{\rm PMNS} \approx \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{6}}\alpha^* \\ \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{2}}\alpha & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}\alpha^* \\ \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{2}}\alpha & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}\alpha^* \end{pmatrix}$$

$\theta_{13} \approx 9^\circ$
requires
$|\alpha| \sim 0.2$

- second column of $U_{\rm PMNS} \propto (1,1,1)^T$
- trimaximal mixing – due to S symmetry

- TB breaking caused by only one free (complex) parameter
- affects θ_{13} and θ_{23} – but not θ_{12}
- get correlations between mixing parameters

$\Longrightarrow$ testable mixing sum rules

$$\theta_{23} \approx 45^\circ - \frac{1}{\sqrt{2}}\,\theta_{13}\cos\delta \qquad\qquad \theta_{12} \approx 35.3^\circ \text{ (TB value)}$$

Flavon alignment

$$\langle \phi_{\mathbf{3}'} \rangle = v_{\mathbf{3}'} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \langle \phi_{\mathbf{2}} \rangle = v_{\mathbf{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \langle \phi_{\mathbf{1}} \rangle = v_{\mathbf{1}} \qquad \langle \widetilde{\phi}_{\mathbf{1}'} \rangle = \widetilde{v}_{\mathbf{1}'}$$

- SUSY unbroken at scale of family symmetry breaking
- F-terms of driving fields $\phi^0_{\mathbf{r}}$ need to vanish

$$\begin{aligned} W_\nu^{\text{flavon}} \quad = \quad & \phi^0_{\mathbf{3}'} (g_1\, \phi_{\mathbf{3}'} \phi_{\mathbf{3}'} + g_2\, \phi_{\mathbf{3}'} \phi_{\mathbf{2}} + g_3\, \phi_{\mathbf{3}'} \phi_{\mathbf{1}}) \\ & + \phi^0_{\mathbf{3}} (g_4\, \phi_{\mathbf{3}'} \phi_{\mathbf{2}}) \\ & + \phi^0_{\mathbf{1}} (g_5\, \phi_{\mathbf{3}'} \phi_{\mathbf{3}'} + g_6\, \phi_{\mathbf{2}} \phi_{\mathbf{2}} + g_7\, \phi_{\mathbf{1}} \phi_{\mathbf{1}}) \\ & + \widetilde{\phi}^0_{\mathbf{1}} (g_8\, \widetilde{\phi}_{\mathbf{1}'} \widetilde{\phi}_{\mathbf{1}'} + M^2) \end{aligned}$$

- yields previously assumed flavon alignments independent of g_i with

$$v_{\mathbf{2}} = -\frac{g_3}{2g_2} v_{\mathbf{1}} \qquad v^2_{\mathbf{3}'} = -\frac{1}{3g_5} \left(g_7 + \frac{g_3^2 g_6}{2g_2^2} \right) v^2_{\mathbf{1}} \qquad \widetilde{v}^2_{\mathbf{1}'} = -\frac{1}{g_8} M^2$$

Parameter counting in neutrino sector

- without family symmetry

$$\left.\begin{array}{ll} Y_\nu & 18 \text{ d.o.f.} \\ M_R & 12 \text{ d.o.f.} \end{array}\right\} \; {\color{red}18} \text{ of which are physical } \left\{\begin{array}{ll} \text{neutrino masses} & 3+3 \\ \text{PMNS mixing} & 3+3 \\ \text{Casas-Ibarra } R & 3+3 \end{array}\right.$$

- in S_4 model

$$\boxed{W_\nu \;\sim\; L N^c H_u + (\phi_{\mathbf{3'}} + \phi_{\mathbf{2}} + \phi_{\mathbf{1}}) N^c N^c + \frac{1}{M} \widetilde{\phi}_{\mathbf{1'}} \phi_{\mathbf{2}} N^c N^c}$$

(i) with arbitrary flavon configurations

$$2 \times (\;1 \;+\; 3 \;+\; 2 \;+\; 1 \;+\; 1\;) \;=\; 16$$

(ii) with flavon alignment

$$2 \times (\;1 \;+\; 1 \;+\; 1 \;+\; 1 \;+\; 1\;) \;=\; 10$$

$$\left.\begin{array}{ll} Y_\nu & 2 \text{ parameters} \\ M_R & 8 \text{ parameters} \end{array}\right\} \; {\color{red}8} \text{ of which are physical}$$

$\longrightarrow$ correlation between low-energy observables guaranteed

Imposing CP symmetry

- naive expectation: all coupling constants real
- closer look at term $\boxed{y\,\psi\,\chi\,\phi}$ in S_4 where
  - (i) $X = \rho(h)$
  - (ii) $\rho^*(h) = X^{-1}\rho(h')X = \rho(\tilde{h})$
  - (iii) Clebsch coefficients are real

$$
\begin{aligned}
y \times c_{ijk}\,\psi_i\,\chi_j\,\phi_k \quad \xrightarrow{\text{CP}} \quad & y \times c_{ijk}\,[X\psi^*]_i\,[X\chi^*]_j\,[X\phi^*]_k \\
&= y \times c_{ijk}\,[\rho(h)\psi^*]_i\,[\rho(h)\chi^*]_j\,[\rho(h)\phi^*]_k \\
&= \left\{y^* \times c_{ijk}\,[\rho^*(h)\psi]_i\,[\rho^*(h)\chi]_j\,[\rho^*(h)\phi]_k\right\}^* \\
&= \left\{y^* \times c_{ijk}\,[\rho(\tilde{h})\psi]_i\,[\rho(\tilde{h})\chi]_j\,[\rho(\tilde{h})\phi]_k\right\}^* \\
&= \left\{y^* \times c_{ijk}\,\psi_i\,\chi_j\,\phi_k\right\}^*
\end{aligned}
$$

- CP maps term $y\,\psi\,\chi\,\phi$ onto its hermitian conjugate if y is real

$\longrightarrow$ naive expectation correct in S_4 (in basis with real Clebsches)

Consequences for the S_4 model

$$M_R = y_{\mathbf{3'}} v_{\mathbf{3'}} \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + y_{\mathbf{2}} v_{\mathbf{2}} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} + y_{\mathbf{1}} v_{\mathbf{1}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + \frac{y_{\mathbf{1'}}}{M} \widetilde{v}_{\mathbf{1'}} v_{\mathbf{2}} \begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

$$v_{\mathbf{2}} = -\frac{g_3}{2g_2} v_{\mathbf{1}} \qquad v_{\mathbf{3'}}^2 = -\frac{1}{3g_5}\left(g_7 + \frac{g_3^2 g_6}{2g_2^2}\right) v_{\mathbf{1}}^2 \qquad \widetilde{v}_{\mathbf{1'}}^2 = -\frac{1}{g_8} M^2$$

- CP symmetry $\rightarrow$ couplings y_i and g_i real
- phases of $v_{\mathbf{1}}$, $v_{\mathbf{2}}$, $v_{\mathbf{3'}}$ identical up to π or $\pm\pi/2$
- absorb phase of $v_{\mathbf{1}}$ into redefinition of N^c

	$v_{\mathbf{1}}$	$v_{\mathbf{2}}$	$v_{\mathbf{3'}}$	$\widetilde{v}_{\mathbf{1'}}$
(A)	real	real	real	real
(B)	real	real	real	imaginary
(C)	real	real	imaginary	real
(D)	real	real	imaginary	imaginary

Predictions with CP symmetry

- seesaw mechanism

$$M_\nu = m_D\, {M_R}^{-1}\, m_D^T \qquad \text{with} \qquad m_D \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\longrightarrow \quad \boxed{U_{\text{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} U_R} \qquad \text{with} \qquad U_R^T\, M_R\, U_R = M_R^{\text{diag}}$$

- resulting PMNS parameters

	θ_{13}	θ_{23}	θ_{12}	δ	(α_1, α_2)
(A)	free	$45^\circ \mp \frac{1}{\sqrt{2}}\theta_{13}$	35.3°	0 or π	0 or π
(B)	free	45°	35.3°	$\pm\frac{\pi}{2}$	0 or π
(C)	unphysical: two degenerate neutrino masses				
(D)	unphysical: $\theta_{13} = 35.3^\circ$				

$\Longrightarrow$ five low-energy predictions with imposed CP symmetry
(up to a finite choice)